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SOME SIMILARITY PROBLEMS OF THE UNSTEADY BOUNDARY LAYER

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UDC 533.6

It was shown in [1] that problems of the nonstationary boundary layer are similarity problems for impulsive motion of an incompressible fluid and motions accelerating with a power law. Some results for an incompressible fluid are presented below.

§1. We consider motion of a semiinfinite flat plate in a compressible liquid, impulsively set into motion. The system of equations for this case [2] is as follows:

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right);$$

$$\frac{\partial\rho}{\partial t}+\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)=0;$$

$$\rho\left(\frac{\partial h}{\partial t}+u\frac{\partial h}{\partial x}+v\frac{\partial h}{\partial y}\right)=\mu\left(\frac{\partial u}{\partial y}\right)^2+\frac{\partial}{\partial y}\left(\lambda\frac{\partial h}{\partial y}\right);$$

$$u=U_e,\ v=0 \quad \text{for} \quad y=0,\ t=0;$$

$$u=0,\ v=0 \quad \text{for} \quad y=0,\ t>0;$$

$$u=U_e,\quad h=h_e,\quad y\to\infty.$$

The notation is conventional; the viscosity μ, the thermal conductivity λ, and the equation of state are all arbitrary functions of temperature and density. We choose

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$$u = \Phi(\xi, \eta),\ v = V(\xi, \eta)/t^{1/2},\ \xi = x/t,\ \eta = y/t^{1/2},\ h = \bar{h}(\xi, \eta),$$

$\rho = \bar{\rho}(\xi, \eta)$, i.e., we convert to a system of similarity variables. Then

$$\bar{\rho}\left[-\xi\frac{\partial\Phi}{\partial\xi} - \frac{1}{2}\eta\frac{\partial\Phi}{\partial\eta} + \Phi\frac{\partial\Phi}{\partial\xi} + \bar{V}\frac{\partial\Phi}{\partial\eta}\right] = \frac{\partial}{\partial\eta}\left[\mu(\bar{T}, \bar{\rho})\frac{\partial\Phi}{\partial\eta}\right];$$

$$-\xi\frac{\partial\bar{\rho}}{\partial\xi} - \frac{1}{2}\eta\frac{\partial\bar{\rho}}{\partial\eta} + \frac{\partial}{\partial\xi}(\bar{\rho}\Phi) + \frac{\partial}{\partial\eta}(\bar{\rho}\bar{V}) = 0;$$

$$\bar{\rho}\left[-\xi\frac{\partial\bar{h}}{\partial\xi} - \frac{1}{2}\eta\frac{\partial\bar{h}}{\partial\eta} + \Phi\frac{\partial\bar{h}}{\partial\xi} + \bar{V}\frac{\partial\bar{h}}{\partial\eta}\right] = \mu(\bar{T}, \bar{\rho})\left(\frac{\partial\Phi}{\partial\eta}\right)^2 + \frac{1}{\sigma}\frac{\partial}{\partial\eta}\left[\mu(\bar{T}, \bar{\rho})\frac{\partial\bar{h}}{\partial\eta}\right].$$

The boundary conditions are

$$\Phi = \bar{V} = 0,\ \ \bar{h} = h_w,\ \ \eta = 0,\ \ \xi > 0,$$
$$\Phi = U_e,\ \ \bar{h} = h_e,\ \ \eta \to \infty.$$

For simplicity, the Prandtl number is assumed to be constant. The possibility of using similarity variables for the arbitrary functions μ, λ and the equation of state arises from the fact that u, h, ρ, for which initial and boundary conditions are given, are not equal to zero and are not explicit functions of time, as v is, for example. In difference form the equations of motion and energy are [we omit the bar above the symbols, $\bar{T} = (T - T_w)/(T_e - T_w)$, and $h = c_p T$]

$$\frac{\Phi_i^{j+1} - \Phi_i^j}{\Delta\xi} = \frac{1}{(U_e\Phi_i^{j+1} - \xi_i)}\left\{\frac{1}{\rho_i^{j+1}}\frac{1}{2\Delta\eta^2}\left[(\mu_{i+1}^{j+1} + \mu_i^{j+1})\Phi_{i+1}^{j+1} -\right.\right.$$
$$\left.\left. - (\mu_{i+1}^{j+1} + 2\mu_i^{j+1} + \mu_{i-1}^{j+1})\Phi_i^{j+1} + (\mu_i^{j+1} + \mu_{i-1}^{j+1})\Phi_{i-1}^{j+1}\right] - \left(V_i^{j+1} - \frac{1}{2}\eta_i\right)\frac{\Phi_{i+1}^{j+1} - \Phi_i^{j+1}}{\Delta\eta}\right\};$$

$$\frac{T_i^{j+1} - T_i^j}{\Delta\xi} = \frac{1}{(U_e\Phi_i^{j+1} - \xi_i)}\left\{\frac{\mu_i^{j+1}(T, \rho)}{c_p\rho_i^{j+1}}\frac{U_e^2}{T_e - T_w}\frac{\Phi_{i+1}^{(j+1)^2} - 2\Phi_i^{j+1}\Phi_{i+1}^{j+1} + \Phi_i^{(j+1)^2}}{\Delta\eta^2} +\right.$$
$$+ \frac{1}{\rho_i^{j+1}\sigma}\frac{1}{2\Delta\eta^2}\left[(\mu_{i+1}^{j+1} + \mu_i^{j+1})T_{i+1}^{j+1} - (\mu_{i+1}^{j+1} + 2\mu_i^{j+1} + \mu_{i-1}^{j+1})T_i^{j+1} +\right.$$
$$\left.\left. + (\mu_i^{j+1} + \mu_{i-1}^{j+1})T_{i-1}^{j+1}\right] - \left(V_i^{j+1} - \frac{1}{2}\eta_i\right)\frac{T_{i+1}^{j+1} - T_i^{j+1}}{\Delta\eta}\right\}.$$

We determine the normal velocity component from the continuity equation by integration. The results of velocity profile calculations are shown in Fig. 1, and compared with an incompressible liquid, for U_e = 2880 cm/sec, T_w = 500°K, T_e = 2400°K, σ = 0.7 [1) ξ = 0.025, compressible gas; 2, 3) incompressible liquid, ξ = 0.05, ξ = 7.55, ν = 0.15; 4) ξ = 8.525, compressible gas, ξ = 21.275 falls on this same curve; and 5) ξ = 7.55, ν = 2.25, incompressible liquid]. A similar picture is obtained for the temperature.

§2. We consider formation of the boundary layer behind a traveling shock wave on a thin semiinfinite flat plate [2]. We denote by θ the rate of propagation of the shock wave and by V, the velocity of the secondary gas flow behind the shock; we reverse the motion in such a way that the shock wave becomes stationary. The boundary-layer region is bounded by $x = \theta t$. The system of equations here is the same as in the previous problem, and the boundary conditions are

$$u = v = 0,\ \ h = h_w,\ \ y = 0,\ \ x > 0,$$
$$u = U_e,\ \ h = h_e,\ \ y = \infty,\ \ x < \theta t,$$
$$u = 0,\ \ h = h_0,\ \ y = \infty,\ \ x > \theta t,$$

where $U_e = \theta - V$.

The enthalpy, velocity, and density at the outer edge are determined by the Hugoniot relations. The results of the computation are shown in Fig. 2, for M_∞ = 3, σ = 0.7, ρ_∞ = 0.00129 g/cm³ [1) ξ = 0.05; 2) ξ = 24.5; 3) ξ = 30.3; 2 and 3 coincide]. The dependence of the dimensionless temperature profile on ξ has the same form.

§3. We consider a semiinfinite flat plate set impulsively into motion in a conducting gas with small magnetic Reynolds number Re_H. A magnetic field of inductance $B_{y,0}$ is applied perpendicular to the direction of motion of the main flow.

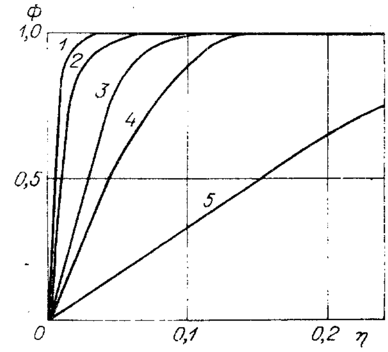

Fig. 1

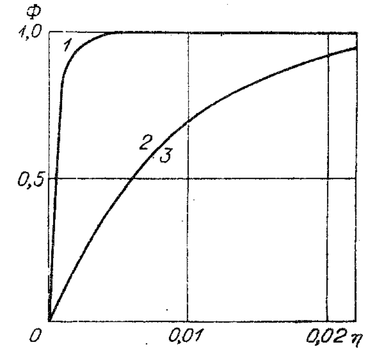

Fig. 2

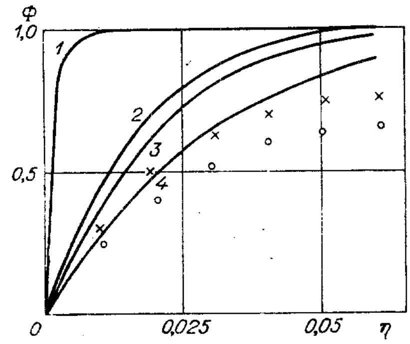

Fig. 3

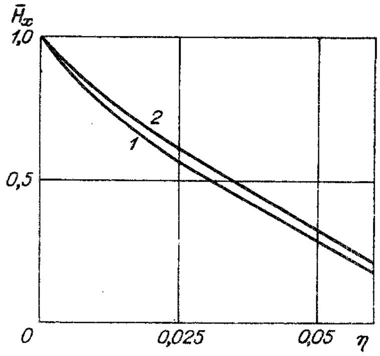

Fig. 4

The equations can be written in the form [3]

$$\frac{\partial\rho}{\partial t}+\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)=0,$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)-\frac{\sigma u}{\rho}B_{y,0}^2,$$

$$c_p\rho\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\frac{\partial p}{\partial t}+u\frac{\partial p}{\partial x}+\frac{\partial}{\partial y}\left(\lambda\frac{\partial T}{\partial y}\right)+\mu\left(\frac{\partial u}{\partial y}\right)^2+\sigma u^2B_{y,0}^2.$$

For simplicity, we consider the equation of state of a perfect gas; for μ, λ, we choose a power dependence on temperature; σ, $B_{y,o}$ are constant. After introducing

$$\rho=\bar\rho(\xi,\ \eta,\ t),\ T=\overline{T}(\xi,\ \eta,\ t),\ u=\Phi(\xi,\ \eta,\ t),$$

$$v=\overline{V}(\xi,\eta,t)/t^{1/2},\quad \eta=y/t^{1/2},\quad \xi=x/t,\quad \overline{T}=(T-T_w)/(T_e-T_w)$$

we obtain

$$-\frac{1}{2}\eta\frac{\partial\rho}{\partial\eta}-\xi\frac{\partial\rho}{\partial\xi}+\frac{\partial(\rho\Phi)}{\partial\xi}+\frac{\partial(\rho v)}{\partial\eta}=0,$$

$$-\frac{1}{2}\eta\frac{\partial\Phi}{\partial\eta}-\xi\frac{\partial\Phi}{\partial\xi}+\Phi\frac{\partial\Phi}{\partial\xi}+V\frac{\partial\Phi}{\partial\eta}=\frac{1}{\rho}\frac{\partial}{\partial\eta}\left[\mu\frac{\partial T}{\partial\eta}\right]-\frac{\sigma\Phi}{\rho}B_{y,0}^2t,$$

$$c_p\rho\left[-\frac{1}{2}\eta\frac{\partial T}{\partial\eta}-\xi\frac{\partial T}{\partial\xi}+\Phi\frac{\partial T}{\partial\xi}+V\frac{\partial T}{\partial\eta}\right]=\frac{\partial}{\partial\eta}\left(\lambda\frac{\partial T}{\partial\eta}\right)+\mu\left(\frac{\partial\Phi}{\partial\eta}\right)^2+\sigma\Phi^2B_{y,0}^2t.$$

The problem is not completely self-similar, but the use of similarity variables allows us to obtain a parametric dependence on time, and the parameter is, in fact, the quantity $\sigma B_{y,o}^2t$. Therefore, a calculation for any one value of this group allows us to obtain a solution for a wide set of values $B_{y,o}$, σ, t. The results of the calculation of velocity are shown in Fig. 3, where 1) ξ = 0.0125; 2) ξ = 1.1375; 3) ξ = 1.7; 4) ξ = 4.5125 for small values of time ($t = 0.01$ sec). The crosses correspond to $t = 0.5$, $\xi = 1.1375$ and the circles, to $t = 0.7$, $\xi = 1.1375$. Initially the curves coincide. We observe a noticeable decrease in friction and heat flux far from the nose, when the magnetic field is present. By solving the problem we can check that the contribution of the time derivatives is small.

§4. We consider the problem where a flat plate begins to move suddenly in a conducting gas at large magnetic Reynolds number Re_H. An electric current passes through the plate perpendicular to the flow (the current is switched on at the time the motion begins and is insulated from the plasma), inducing a magnetic field in the plasma. When the incident flow plasma has high conductivity, the magnetic field due to currents within the wall is localized in a region close to the wall surface.

The equations of the dynamic, thermal, and magnetic boundary layers have the form [3]

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0;$$

$$\frac{\partial H_x}{\partial t} + u\frac{\partial H_x}{\partial x} + v\frac{\partial H_x}{\partial y} + H_x\frac{\partial v}{\partial y} - H_y\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(\nu_H\frac{\partial H_x}{\partial y}\right);$$

$$\frac{\partial H_x}{\partial x} + \frac{\partial H_y}{\partial y} = 0;$$

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) - \mu_e H_y\frac{\partial H_x}{\partial y} = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right);$$

$$p + \frac{1}{2}\mu_e H_x^2 = p_0(x);$$

$$\rho\left(\frac{\partial h}{\partial t} + u\frac{\partial h}{\partial x} + v\frac{\partial h}{\partial y}\right) = \frac{\partial p}{\partial t} + u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + \frac{\partial}{\partial y}\left(\lambda\frac{\partial T}{\partial y}\right) + \mu\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\nu_H}{4\pi}\left(\frac{\partial H_x}{\partial y}\right)^2.$$

Here we use the Gaussian system of units, and the notation is conventional. As before we seek a solution in the form

$$\rho = \bar{\rho}(\xi, \eta),\ h = \bar{h}(\xi, \eta),\ u = \Phi(\xi, \eta),\ H_x = \bar{H}_x(\xi, \eta),$$
$$H_y = \bar{H}_y/t^{1/2},\ \text{where}\ \xi = x/t,\ \eta = y/t^{1/2}.$$

We omit the bar above the symbols ($h = c_pT$). After substituting in the equation, we obtain

$$-\xi\frac{\partial \rho}{\partial \xi} - \frac{1}{2}\eta\frac{\partial \rho}{\partial \eta} + \frac{\partial(\rho\Phi)}{\partial \xi} + \frac{\partial(\rho V)}{\partial \eta} = 0;$$

$$-\xi\frac{\partial H_x}{\partial \xi} - \frac{1}{2}\eta\frac{\partial H_x}{\partial \eta} + \Phi\frac{\partial H_x}{\partial \xi} + V\frac{\partial H_x}{\partial \eta} + H_x\frac{\partial V}{\partial \eta} - Hy\frac{\partial \Phi}{\partial \eta} = \frac{\partial}{\partial \eta}\left(\nu_H\frac{\partial H_x}{\partial \eta}\right);$$

$$\frac{\partial H_x}{\partial \xi} + \frac{\partial H_y}{\partial \eta} = 0;$$

$$p_0 = p + \frac{1}{2}\mu_e H_x^2 = \text{const};$$

$$\rho\left(-\xi\frac{\partial \Phi}{\partial \xi} - \frac{1}{2}\eta\frac{\partial \Phi}{\partial \eta} + \Phi\frac{\partial \Phi}{\partial \xi} + V\frac{\partial \Phi}{\partial \eta}\right) - \mu_e H_y\frac{\partial H_x}{\partial \eta} - \mu_e H_x\frac{\partial H_x}{\partial \xi} = \frac{\partial}{\partial \eta}\left(\mu\frac{\partial \Phi}{\partial \eta}\right);$$

$$\rho\left(-\xi\frac{\partial h}{\partial \xi} - \frac{1}{2}\eta\frac{\partial h}{\partial \eta} + \Phi\frac{\partial h}{\partial \xi} + V\frac{\partial h}{\partial \eta}\right) = -\xi\frac{\partial p}{\partial \xi} - \frac{1}{2}\eta\frac{\partial p}{\partial \eta} + \Phi\frac{\partial p}{\partial \xi} + V\frac{\partial p}{\partial \eta} + \frac{\partial}{\partial \eta}\left(\lambda\frac{\partial T}{\partial \eta}\right) + \mu\left(\frac{\partial \Phi}{\partial \eta}\right)^2 + \frac{\nu_H}{4\pi}\left(\frac{\partial H_x}{\partial \eta}\right)^2$$

or

$$\rho c_p\left(-\xi\frac{\partial T}{\partial \xi} - \frac{1}{2}\eta\frac{\partial T}{\partial \eta} + \Phi\frac{\partial T}{\partial \xi} + V\frac{\partial T}{\partial \eta}\right) = -\xi\frac{\partial p}{\partial \xi} - \frac{1}{2}\eta\frac{\partial p}{\partial \eta} + \Phi\frac{\partial p}{\partial \xi} + V\frac{\partial p}{\partial \eta} + \frac{\partial}{\partial \eta}\left(\lambda\frac{\partial T}{\partial \eta}\right) + \mu\left(\frac{\partial \Phi}{\partial \eta}\right)^2 + \frac{\nu_H}{4\pi}\left(\frac{\partial H_x}{\partial \eta}\right)^2.$$

We use an implicit difference scheme to solve the problem. Two blocks were isolated in the calculation. In the first block we simultaneously use the method of matrix forcing to solve the equations of motion and energy until the iterative process is completed. In the second block we find the magnetic field intensity profiles. The solution process in layer ξ_i is considered to be ended when all the profiles have been calculated with a given accuracy. The accuracy must be kept high, to avoid a substantial accumulation of errors with increase of ξ. The distribution of magnetic field intensity H_x is given in Fig. 4 [1) $\xi = 0.0125$; 2) $\xi = 4.137$, $U_e = 0.2 \cdot 10^5$ cm/sec]; and μ, λ, ν_H are considered to be power series functions of the density and temperature.

The above program was chosen because of the capabilities of the M-222 computer memory. It may seem that the use of matrix forcing for simultaneous computation of all the desired quantities would lead to more rapid solution of the problem. It should be noted that, while the accuracy of the increase of the vertical velocity component is not important in calculating the unsteady boundary layer for an incompressible liquid, and the results vary only by a few percent when this is totally omitted; nevertheless, this component requires accurate computation for a compressible liquid.

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INTERNAL RESONANCES IN HYDRODYNAMICS

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§1. In the theory of the vibrations of systems close to linear [1], internal resonance is defined as the proportionality of several natural frequencies to natural numbers. The present article discusses internal resonances in hydrodynamics.

In the case of internal resonance, forced vibrations of small amplitude, brought about by a harmonic perturbation, can differ considerably from harmonic. An example is discontinuous vibrations of a gas (shock waves), observed in a closed tube with a harmonic motion of a piston [2, 3].

Autovibrations of small amplitude can also be essentially nonharmonic, for example, autovibrations in a low-pressure gas discharge [4].

The main features of resonances come out with the consideration of the boundary-value problem for the real vector X:

$$\frac{\partial X}{\partial t} + L_1 X + L_2 X^2 + \ldots = \sum_{k=1}^{n} \varepsilon_k C_k e^{i\omega_k t} + \text{ c.c.}, \quad UX = 0 \tag{1.1}$$

(c.c. is an expression, complex-conjugate to the preceding). Here the real coefficients L and the matrix U in the boundary condition can depend on the coordinates x and are polynomials with respect to $D = \partial/\partial x$. The region of change of x is assumed to be bounded. Each perturbation with the frequency $\omega_k > 0$ and the form $C_k(x)$ is proportional to a small amplitude ε_k. The frequencies ω_k and their differences are assumed to be fairly great (the effects of the type of slow change in the parameters are not taken into consideration here). It is postulated that the problem

$$pX + L_1 X = 0, \ UX = 0 \tag{1.2}$$

has several simple eigennumbers $p = \gamma + i\Omega$, with small increments of γ and positive frequencies Ω. Let these be the numbers p_m ($m = 1, 2, \ldots, M$), the corresponding eigenfunctions

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